The Abacus and the Slave Market

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As is well known, references to a new abacus using counters with inscribed numerals turn up in Latin medieval sources around the year 1000. The earliest certain evidence for its use is Gerbert's teaching of it in the cathedral school in Rheims (where he taught from 973 onward [Riché, 1985: 55]) as described by his biographer Richer [ed. Bubnov, 1899: 381].

On the faith of later descriptions of the device it has been the normal assumption that these inscribed numerals (called *caracteres* by Richer, also called *apices* for instance in *Boethii quae dicitur geometria altera* [ed. Folkerts, 1970: 139]) were *ghubār* numerals (the Western form of the Hindu-Arabic numerals) from the very beginning. In [1985: 199–212], however, Werner Bergmann claimed that the *ghubār* numerals first came into use during the eleventh century, and that Gerbert and his contemporaries had marked their counters with Greek alphabetic numerals. All early descriptions of the abacus that mention the *caracteres* except Bernelinus (a probable student of Gerbert) fail indeed to explain which numerals were used – see [Folkerts, 1996: 24–26]. Bernelinus, on his part, presents both the *ghubār* and the Greek possibility [ed. Olleris, 1867: 361]. Since Bernelinus refers in his introduction to Gerbert as *papa* [ed. Olleris, 1857: 357], his treatise must postdate 999; Bergmann [1985: 199 n. 167] makes the observation that the reference has a form that would be used about still living persons, which Gerbert was no longer in 1004; but he immediately discards this dating as a mere hypothesis.

As long as the earliest certain evidence for use of *ghubār* numerals was the compilation *Boethii quae dicitur geometria altera* (to be dated to the second quarter of the eleventh century), this seemed no impossible reinterpretation of the evidence, and it convinced scholars like Eberhard Knobloch [1987: 151] and Charles Burnett [1996: 227]. However, as objected in [Folkerts, 1996: 27–33], in two illustrations of

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abacus treatises that correspond quite precisely to what is said by Richer and Bernelinus, every third column is numbered with *ghubār* numerals;[1] one appears to belong to the beginning of the eleventh century, the other may even be from the late tenth. Moreover, in [2002] Burnett discovered two similarly dated abacus specimens (not illustrations) that also correspond very precisely to Richer's description of Gerbert's device and to the description given by Bernelinus and carrying *ghubār* numerals in the same way; other evidence also connects them to Gerbert's immediate surrounding.

This leaves no reasonable doubt that the use of *ghubār* numerals on the counters is original or at least that they were used by Gerbert; the Greek letter-numerals may be an innovation introduced later by scholars who knew about these, for instance from the presentation of finger-reckoning in *De temporum ratione* [ed. Jones 1943: 181].[2] So, it would seem reasonable to return to the traditional assumption, for instance as formulated by Burnett [2002: 92],

> that Gerbert introduced the practice of marking the counters with Arabic numerals (which he would have come across when he studied in Catalonia, before coming to Reims), and established a form of the abacus board that became an exemplar for most subsequent teachers of the abacus.

One detail, however, may raise doubts at least about the Catalan hypothesis. The *caracteres* carry names:[3]

1	2	3	4	5	6	7	8	9
igin	andras	ormis	arbas	quimas	calcis	zenis	temenias	scelentis

At first – that is, until the 1080s – the names are not used in texts describing the use of the abacus but only in depictions of the board and various other texts [Folkerts, 2000: 219–230; 2003: 12–14] – in particular in the second manuscript mentioned in note 1 and in a poem also accompanying one of the abacus specimens

[1] Paris, Bibliothèque Nationale, lat. 8663; and Codex Vaticanus latinus 644. Both are reproduced in [Folkerts, 2003: Fig. 5-7].

[2] Since the following generations of writers on this abacus all ascribe it to Gerbert, and since the use of marked counters appears to be a more decisive step than the introduction of a set of unfamiliar symbols, it seems likely that the introduction of letter-numerals postdates Gerbert's teaching – but we cannot be sure.

[3] In this spelling in an illustration to the *Boethii quae dicitur geometria altera*, see [Folkerts, 1970: 13].

discovered by Burnett, which makes it likely that the names were used in Gerbert's surrounding.

It has often been observed that some of these names go back to Semitic languages – see, for instance, [Bubnov, 1914: 68] and [Bergmann, 1985: 175 n. 4] reporting observations made by Paul Kunitzsch. Various fanciful attempts were made in the nineteenth century to trace others to Assyrian (actually, also a Semitic language) or Greek. These are listed in a little noticed article by Fr. Th. Köppen from [1892], published in *Bulletin de l'Académie des Sciences de St. Petersbourg* (which may explain why the article has largely gone unnoticed[4]) and also in [Folkerts, 2000: 217–219; 2003: 10*f*]. Moritz Cantor mentions Köppen's work briefly [1907: 895] but only along with the Greek and Assyrian etymological fantasies, probably because its outcome seems to make no sense.

Such an attitude would in any case be understandable. What Köppen finds is the following:

– *igin* is of Berber origin (< *ighem*/*iggen*). Independently, Kunitzsch as reported in [Folkerts, 2003: 11] makes the same observation;

– *andras* is nothing but Old High German (Köppen mentions only German, but Old High German goes as well);

– *ormis* seems to be related to Magyar, where 3 is *három*, with stem *harm*;

– *arbas* is Semitic 4 (Arabic *arba'*a);

– *quimas* is supposed to be of Latin origin – Köppen points to forms like *quinus* and *quini*;[5]

– *calcis* (sometimes written *caltis*, *calctis* or *chalcus*) presents problems. Köppen points to frail similarities with Chechen and Turkish but disregards them as far-fetched. A correspondent of his (N. Anderson) proposes the original spelling to be possibly *cattis*, which would point to Old Magyar **kăted* or **kătid* (reconstructed from Magyar hatod and Vogul *katit*), which makes somewhat more sense;

[4] Bubnov [1914: 65 n. 4] knows about it but says that he did not have access to it.

[5] Kunitzsch (in [Bergman 1985: 175 n. 4]) proposes another possibility: a derivation from Arabic *khamsa*, via Spanish orthography (which hardly existed in the tenth century – but Latin might easily transcribe /*kh*/ in this way).

– *zenis* is also left unexplained;

– *temenias* (sometimes written *zemenias*) is an obvious loan from Arabic *thamāniya*, 8 (or from another Semitic language);

– *celentis* seems to be derived from Magyar *kilencz*, meaning 9, presumably **kilentíz* in Old Magyar.

Where would all these languages – Arabic, Berber, Magyar, High German and perhaps Latin – have come together? Within the traditional paradigm for our discipline, where mathematics comes from "mathematicians" (when innovative) or from earlier mathematical writings (when less innovative) it is difficult to answer. Köppen suggests that German missionaries and merchants went to Hungary after the Christianization of that area in 973 and brought back some number words, which because of their exotic character were then adopted. But there is another possibility. Before 973, it may not have been safe for German merchants to go to Hungary, but Hungarian slave traders certainly came to the slave markets in Verdun and elsewhere in Lorraine,[6] where they sold the human cattle they had captured or bought in Slavonic area[7] to Arabic (and probably Berber) colleagues. Here, there will have been no need of exoticism: sales and prices will have been negotiated in a mixture of all the merchant languages involved: Magyar, Arabic, Berber, High German, and perhaps some Latin. And Lorraine was the homeland of the new abacus, as stated by Richer [ed. Bubnov, 1899: 383] and confirmed by the distribution of early treatises and specimens.

Strictly speaking, this only concerns the names, not the *ghubār* numerals, but in view of the Arabic origin of the latter as well as some of the former it is likely that the two went together, and that whoever started using *ghubār* numerals on the abacus knew them from the local slave market and was in no need to have been to Catalonia.[8]

[6] On the central role of Verdun, cf. [*Cambridge Economic History of Europe* II, 211, 322, 417 and 486] and [Verlinden, 1970: 8*f*].

[7] And not only there: in 954, "a girl of noble family, captured in the outskirts of Worms, was put up for sale in the city" [Bloch, 1965: I, 11].

[8] The import of the names *could* have taken place already in the later ninth century, since the trade with Slavonic slaves goes back to that epoch [Verlinden, 1970: 4]. The appearance of the names in a praise of music ascribed to the late ninth-century Benedictine monk Bertrandus (or Bernardus) Prudentius but known only from much later manuscripts (twelfth and fourteenth centuries), if true, would exclude the Catalan import but not the role of the slave trade. See https://www.musicologie.org/Biographies/b/bertrandus.html (accessed 13 June 2023).

How (if this hypothesis holds true) these number words made it from the slave market to the abacus teaching of the cathedral schools of Rheims and other bishoprics of Lorraine is a question yet to be settled – probably never to be settled. In particular it will probably remain undecidable whether Arabic traders used a reckoning board with counters carrying inscribed numerals like the one described by al-Uqlīdisī in 952 in Damascus [ed. Saidan 1978: 313*f*].

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